The collective dynamics of human crowd motion: Where physics meets cognitive science

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Introduction

Human crowds display a rich variety of collective behaviours that support an efficient motion under everyday conditions (Helbing *et al.*, 2001). For example, when two flows of people are moving in opposite directions in a crowded street, pedestrians spontaneously share the available space by forming lanes of uniform walking directions, as illustrated figure 1. This “pedestrian highway” is a decentralized collective organization that enhances the traffic efficiency by reducing the need for avoidance manoeuvres. As another example, when two opposite flows meet at a narrow bottelneck, each flow temporaily “captures” the doorway during a short time period, resulting in alternating bursts of pedestrians passing first in one direction and then in the other. Again, this spontaneous group coordination allows for an efficient usage of congesion areas (Helbing *et al.*, 2005).

This surprising ability of pedestrian crowds to solve coordination problems without external control or centralized planning is often referred to as the “*wisdom of crowds*” (Surowiecki, 2004). Indeed, individuals composing the crowd do not actively seek such collective organizations. Instead, each pedestrian simply tries to reach its own destination and walk at its own confortable walking speed. Moreover, most of them are not even aware of the collective organization they participate in. Yet, the crowd as a whole finds efficient solutions to coordination problems.

Interestingly, similar observations have been made in the field of collective animal behaviours (Camazine *et al.*, 2001). For example, when a colony of ants *Eciton Burchelli* is foraging to a food source, the bidirectionnal traffic that establishes along the foraging trail is also organized into lanes, which increases the foraging efficiency (Couzin & Franks, 2003). Another well-known example is the amazing ability of starling flocks and fish schools to coordinate their movements when avoiding predator strikes (Ballerini *et al.*, 2008). In all these systems, as well as in human crowds, there exists a remarkable contrast between the group’s ability to solve coordination problems at the global scale and the limited information and processing time of each group members.

Previous research have shown that various biological and social systems, including ant colonies, bird flocks and human crowds, are actually driven by similar mecanisms of self-organization(Sumpter, 2006; Couzin & Krause, 2003; Ball, 2004). Self-organized systems are decentralized, that is, each individual behaves according to its own motivations and does not possess any global information of the group’s behaviour. However, each of them is strongly affected by the behaviour of his neighbours. For example, when a fish in a school perceives a danger and suddenly change its swimming direction, the neighbouring fish will tend to imitate it. Similarly, if a few pedestrians in a crowd need to reduce their walking speed to avoid an obstacle, those behind them will slow down in turn. Therefore, the

Figure 1: Illustration of the spontaneous organization of pedestrian flows. People walking in opposite direction share the available space by forming lanes.

whole group constitutes a large and complex network of interactions where local information propagates rapidly from one individual to another. In such a way, small perturbations are often amplified at the group level, leading to the emerge of global collective behaviours, such as the traffic organization of pedestrian flows into lanes.

The combinaison of local interactions, however, does not always generate efficient solutions at the group level. In human crowds, for example, it has been shown recently that above a critical density value the collective coordination suddenly breaks down and lets place to a phenomenon called crowd turbulence, where the flow of pedestrians becomes unstable (Helbing *et al.*, 2007). This particular regime is often observed during crowd disasters and is characterized by random and unintended displacements of people in all possible directions. This phenomenon constitutes another example of collective behaviours that emerge out of local interactions among pedestrians, but resulting in unadapted and harmful dynamics.

In this context, the aim of my doctoral thesis is to better understand the behavioural mecanisms at the origin of the dynamics of human crowds motion. For this, it is necessary to understand precisley how pedestrians behave, what kind of information they use from their neighborhood, and how they adapt their behaviour according to these cues.

In current litterature, the dynamics of human crowds is often studied through analogies with physical systems. In fact, the motion of crowds can often be compared to a fluid or a gas as shown by Leroy Henderson (Henderson, 1971, 1974). More recently, various models of pedestrian behaviour based on analogies with Newtonnian force models have been elaborated (Hoogendoorn, 2004; Yu *et al.*, 2005; Steffen, 2008). The *social force model*, for example, suggests that the motion of a pedestrian can be described by the combination of a driving force, that reflects the pedestrian's internal motivation to move in a given direction at a certain speed, and repulsive forces describing the effects of interactions with other pedestrians and boundaries such as walls or obstacles in streets (Helbing, 1995). In particular, it has been shown that force-based models are able to reproduce some crowd observations quite well, such as the emergence of lanes in bidirectionnal traffic. These models, however, are often relatively hard to calibrate and, most of the time, lack quantitative experimental validation. In other words, the precise mecanisms at the origin of collective pedestrian behaviours remain poorly understood.

In this research, we have studied the features of pedestrian interactions to get a quantitative understanding of crowd dynamics. For this, we first conducted a series of laboratory experiments to measure precisely how pedestrians interact with each other during avoidance maneuvers. In particular, we studied the role of social conventions during pedestrian interactions. Then, we focused on social interactions among pedestrians walking in groups, such as a few friends walking together. Based on empirical data, we demonstrated the crucial role of social behaviours in the dynamics of pedestrian crowds. The third section is dedicated to the elaboration of a new modeling framework based on concepts from cognitive science. Finally, I will summarize and discuss these results in the last section of this article.

Experimental measurements of pedestrian interactions

When studying complex dynamics in animal groups, biologists often rely on a typical experimental approach described in Camazine *et al.* (2001). This approach focuses on the features of the interaction laws among individuals, which is a key element to understand the emergence of collective behaviours. The first step consists in measuring under controlled laboratory conditions how an individual adapts his behaviour in the presence of another individual. Then, based on these observations, an interaction model can be elaborated. Finally, numerical simulations of this model in a collective context are compared to empirical observations in the same conditions for a final validation step. As human crowds and animal swarms are driven by similar processes of self-organization, we have applied this approach to the behaviour of pedestrians (Moussaïd *et al.*, 2009).

The experiment we conducted took place in an experimental corridor of length l=8m and width w=2m. Fourty participants were tracked while walking in the corridor under two simple conditions: first in the absence of interactions, and then in the presence of another pedestrian moving in the opposite direction. Both conditions have been replicated approximately 150 times, and pedestrians' movements were analyzed by means of a video tracking system.

By comparing the speed and acceleration of participants in both conditions, it has been possible to measure precisely how the interaction has affected the behaviour in terms of walking speed and direction changes. In particular, our observations revealed that pedestrians have a strong tendency to avoid each other toward the right-hand side (i.e. more than 80% of the observations).

Based on the above quantitative measurements, we derived the precise mathematical function describing how a pedestrian adapts his behaviour during an interaction. This interaction law was then implemented in a new specification of the social force model. In particular, the interaction function has been fitted on our observations, and includes a directional biais to account for the side preference. Then, we compared the predictions of this new model with empirical observations in a collective context. For this, we made a series of video recordings of pedestrian traffic in a pedestrian walkway during a crowded afternoon in the city of Bordeaux (France). In parrallel, we conducted computer simulations of the new model under the same conditions. In both empirical data and computer simulations, we found that opposite flows of pedestrians spontaneously organise into lanes. Moreover, observed and simulated patterns exhibited a pronounced left/right asymmetry, where lanes appeared most of the time on the right-hand side of the street, with respect to the walking direction (see for example figure 1). It appeared that the side preference facilitates coordination when two pedestrians meet each other head on. After a short transition time, a few pedestrians end up walking one behind the other on the right-hand side of the street, which is a stable walking position that does not require further avoiding manoeuvres. Moreover, it becomes increasingly difficult to walk in front of the opposite flow, which increases the separation of the flows. Finally, this results in the observed asymmetric traffic organization.

While the side preference plays an important role in the traffic organization, the origin of this behavioural biais remains unclear. In France (the place where our experiments took place), pedestrians exhibit a preference for the right-hand side. In other geographical areas, however, people exhibit left-hand traffic organisation, such as several Asian countries. In some areas of Great Britain, it has been reported that pedestrian traffic also set up on the right-hand side (Helbing *et al.*, 2001). Therefore, it seems the side preference is not directly coupled to the direction of car traffic rules. Instead, this behavioural biais can be interpreted as a *social convention* that sets up spontaneously in a population where it is more efficient for each individual to behave like the majority (Arthur, 1990). In fact, numercial simulations have shown that a simple learning rule, where individuals tend to reproduce previous successful strategies, is sufficient to generate a social convention for the same avoidance side (Helbing, 1991). Therefore, the social convention helps the crowd solving coordination problems, while it reduce the cognitive effort of choosing a side at each new interaction.

In summary, we relied on an experimental approach inspired from the study of animal behaviour to caracterize the laws ruling pedestrian behaviour during interactions. This resulted in the elaboration of a model built from the bottom-up and based on experimental measurements only. Therefore, the interaction function is not just chosen in a plausible way, but it explicitly represents experimentally determined features of pedestrian behaviour. In such a way, it has been possible to highlight the existence and the role played by the side preference, which we interpreted as a social convention that helps the crowd solving coordination problems.

Social interactions in traffic flows

Interactions among pedestrians does not only consist is avoiding each other. In 1961, the social scientist James Coleman has reported a series of observations showing that most people in crowds are not isolated pedestrians, but part of small social groups, such as a few friends walking together (Coleman & James, 1961). In this case, the nature of interactions among group members differs from the avoidance interactions investigated before. However, Coleman's observations were rarely taken into account in the study of crowd dynamics, and a quantitative analysis of such social interactions has never been undertaken. In this section, I will describe the work we did to investigate on the features of social interactions among pedestrians walking in small groups (Moussaïd *et al.*, 2010). For this, we have analyzed the behaviour of approximately 1500 pedestrian groups through video recordings of crowds in public areas.

First, the analysis confirmed previous observations by Coleman, that is, pedestrians rarely walk alone. In fact, we found that 50 to 70% of pedestrians in crowd are actually moving in small groups composed of two to four members. Then, we analyzed the spatial organisation of these groups at different density levels. At low density, we found that group members tend to walk side-by-side, forming a line perpendicular to the walking direction. This configuration facilitates social communication

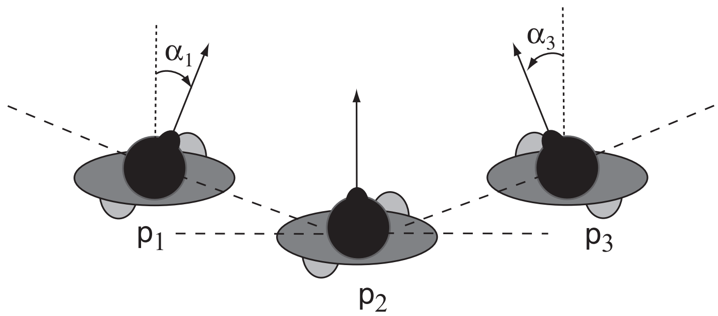

Figure 2: In the model, each group member i turns his head by angle angle α_i in order to have his partners included in his vision field (represented by the dashed lines). The 'V'-like formation illustrated above constitutes a stable walking configuration, where each pedestrian keeps a visual contact with his partners.

within the group, since each member can easily communicate with his partners without turning the back to any of them. However, this walking pattern also occupies a large area in the street. As the density increases, the available space around the group is reduced and the group configuration changes: From a side-by-side walking pattern, groups composed of three and four members turned to a 'V'-like formation, where the pedestrians in the middle of the group tended to stand back while those on the side got closer to each other (Figure 2). From a physics-based point of view, the 'V'-like pattern is surprizing. Indeed, in physical systems, a flexible structure moving against an opposite flow is bent backward forming a *reverse* 'V' configuration, which has optimal aerodynamics features. This holds also for moving animal groups, as shown by the *reverse* 'V' flying configuration of gees flocks. On the other hand, the observed 'V'-like configuration facilitates social exchange and support a better communication among group members. Therefore, it seems that group members actively resist to the physical constraints induced by other pedestrians to facilitate social exchanges with their partners.

To better understand these walking patterns, we developed a model of group behaviour. Early research in psychology and social science have shown that social interactions are mainly characterized by verbal exchanges and visual contacts among partners (Kendon, 1967; Argyle & Dean, 1965). Accordingly, we have extended the model elaborated in the previous section to include social interaction effects. For this, we introduced a representation of the vision field of pedestrians. Then, we assumed that each group member rotates his head to have a visual contact all his partners (figure 2). However, the greater the head rotation angle, the less confortable is the walking position. Therefore, pedestrians tend to adjust their position to reduce the head rotation. In addition, pedestrians try to keep a certain distance to the group centre of mass, to allow for verbal exchanges. Finally, the effects of social interactions were cumulated with out-group avoidance interactions.

Numerical simulations of this new model predict walking patterns in agreement with those observed, where group members walk side-by-side at low density, and organize into a 'V'-like configuration as the density increases. Moreover, the model reveals that pedestrian groups have a significant impact on the traffic efficiency. It turned out that the 'V'-like walking configuration combined to the ubiquity of pedestrian groups in crowds leads to a reduction of the walking speed by an average of 17% as compared to a situation with isolated pedestrians only.

This work highlights the interplay between social interactions among group members and avoidance interaction with out-group pedestrians. Both social and avoidance interactions contribute to solving coordination problems: While avoidance interactions shape the traffic flow by generating lanes (as described in the first section), social interactions generate walking patterns that facilitate communication among a group members. Interestingly, social and avoidance interactions have confliting effects. Indeed, social interactions produce a side-by-side configuration where pedestrians form a line *perpendicular* to the walking direction. In opposite, avoidance interactions generate lanes of people walking one behind each other, that is, forming lines *parrallele* to the walking direction. Yet, the amount of avoidance interactions increases with the density while the amount of social interactions remains the same as long as the group size is unchanged. Consequently, the walking configuration of pedestrian groups changes continuously with increasing density ranging from a side-by-side configuration at low density to a pattern where they move one behind the other in crowded situations, through a 'V'-like configuration at intermediate densities.

In summary, these insights demonstrate that crowd dynamics is not only determined by physical constraints induced by other pedestrians and the environment, but also significantly by communicative and social interactions among individuals.

Modeling pedestrian behaviour through cognitive heuristics

In the previous sections, we relied on attraction and repulsion forces to formalize our observations. The strength and direction of these forces implicitly reflected the result of two cognitive processes: the sensory perception of the environment and the cognitive processing of this information. However, this required the use of sophisticated mathematical expressions that were often hard to calibrate. In this section, we suggest to replace the typical force-based description by a cognitive model of pedestrian behavior. For this, two crucial questions need to be adressed : " *What kind of information is used by pedestrians ?*", and " *How this information is processed to adapt the walking behaviour ?*". As already suggested in our previous work, vision seems to be the most important information source used by pedestrians. In the field of behavioural ecology, early works by James Gibson already investigated how the motion of a living-being is related to its visual perception of the surrounding environment (Gibson, 1958). Based on Gibson's theories, various research have shown that pedestrians actually rely on specific visual cues when they navigate in public buildings, such as the length of the sight lines or the coverage of their vision field (Turner & Penn, 2002; Hillier & Hanson, 1984). In these work, however, only interactions between a pedestrian and his environment have been investigated, while interactions *among* people are the most important for crowd dynamics.

Possible answers to the second question are suggested in the field of psychology and cognitive science. It has been shown that people often rely on fast and simple cognitive procedure - called *heuristics* - to quickly adapt their behaviour in situations where they have to cope with complex and rapidly changing environments (Gigerenzer & Todd, 1999).

Here, we propose a new modeling framework for crowd behaviour, based on the idea that pedestrians apply cognitive heuristics based on their visual perception of the surrounding environment. For this, we first define a representation of the pedestrian's visual input $f(\alpha)$, representing the distance the pedestrian can walk in the direction α before a collision with another pedestrian or obstacle occurs. Here, α is varied from the one boundary of the vision field to the other (Figure 3). Based on this function, we define two behavioural heuristics to describe how the pedestrian adapts his walking direction α_{des} and walking speed v_{des}. First , the pedestrian choose the direction α_{des} of the empty space that allows him to come closest to his destination point. This can be easily computed given the visual informations defined by $f(\alpha)$. In such a way, the pedestrian tries to minimizing the visual

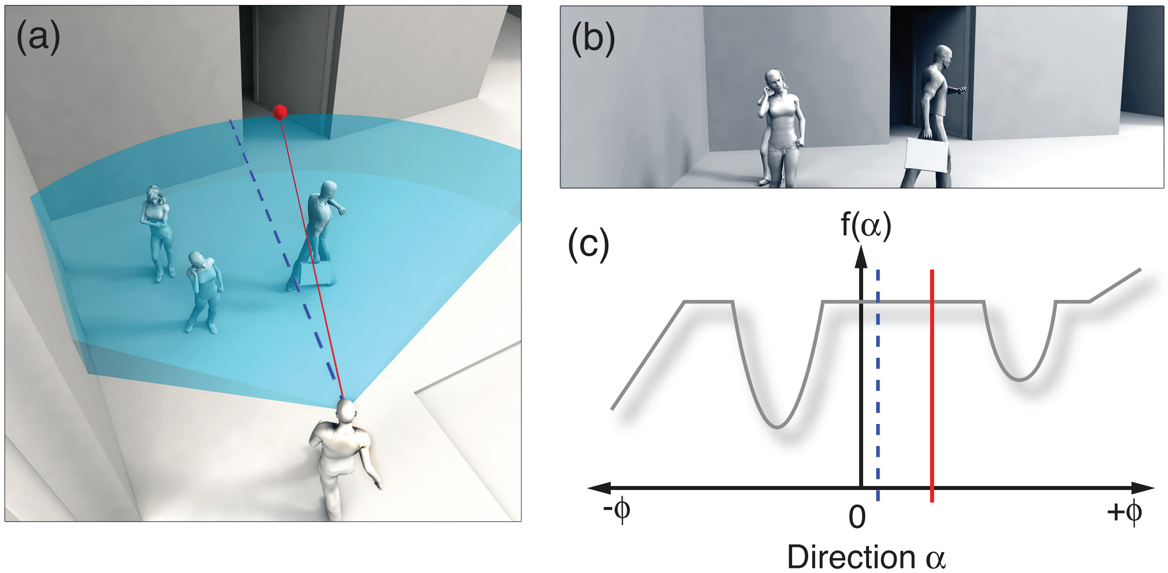

Figure 3: (a) Illustration of a pedestrian facing three other individuals and trying to reach a destination point marked in red. The blue dashed line represents the looking direction and the transparent blue area is the vision field of the pedestrian ranging to the left and to the right of the looking direction by an angle ϕ. (b) The same situation as seen by the pedestrian. (c) Graphical representation of the function $f(\alpha)$ describing the distance before collision in all visible directions, taking into account the other individuals' walking speed and walking direction.

coverage in the walking direction, while not deviating too much from his destination. Second, the pedestrian keeps a time to collision at least equals to his reaction time τ, with respect to the first obstacle in the direction α_{des}. In other words, the pedestrian continuously adjust his walking speed v_{des} to ensure a sufficient time for stopping in case of unexpected encounter. Finally, at each time step the pedestrian adapts his current walking speed and direction to the desired ones within a time period τ. The reaction time τ, therefore, is the only parameter of the model and has been measured during our previous laboratory experiments to τ =0.5s .

One fundamental novelty of this approach is that pedestrians *seek a free way* though the crowd, instead of being *repelled* by their neighbors as it was assumed in previous force-based models. In simulations, the heuristic-based model predicts individual pedestrian behaviour well. In particular, it predicts avoidance trajectories in basic interaction situations in quantitative agreement with the laboratory experiments described in section 1. In a collective context, the model also predicts the emergence of collective patterns, such as the formation of lanes in bidirectionnal flows.

However, it appeared that heuristics were not sufficient at very high densities, when body contacts occur between neighbouring pedestrians. Indeed, when people are very densely packed, the movements of pedestrians become *unintentional*: That is, it results from the physical pressure exerted by neighbouring individuals. Therefore, when body contacts occur in the crowd, *physical interactions* also come into play. Accordingly, we have coupled the above heuristic-based navigation model with repulsive forces addressing body contacts at high density. The physical repulsion forces are defined according to the body pressure value and reduced to zero if the pedestrians do not touch each other.

The combinaison of cognitive heuristics and repulsion forces allows for simulations of high density situations. Above a certain density level, physical interactions start to dominate over the heuristic-based behaviour. Then, intentional movements of pedestrians are increasingly replaced by unintentional mass motion, leading to the emergence of crowd turbulence around bottleneck areas. Further analysis shows that areas of strong body compression produce sudden stress release and earthquake-like mass displacement of people in all possible direction. Then, a shock wave propagates to the neighbouring pedestrians, causing a global loss of coordination.

Discussion & Conclusion

Under everyday conditions, human crowds display a rich variety of self-organized behaviours to overcome coordination problems. At extreme densities, however, coordination can suddenly break down causing serious accidents at mass events. During this thesis, I have investigated the mecanisms underlying such collective behaviours. By combining laboratory experiments, analysis of video recordings in public places, and mathematical modeling, I studied how interactions among pedestrians could lead to such a variety of collective phenomena.

The first research that has been overtaken is a precise analysis of pedestrian interactions in laboratory conditions. Based on an experimental approach typical from swarm biology, we highlighted the existence of a social convention for avoiding others toward the right-hand side. In particular, numerical simulations demonstrated that this bias facilitates coordination among pedestrians and shapes the traffic organization at the crowd level. Then, we studied social interaction among people walking in group. Through analysis of video recordings, we have demonstrated that crowd dynamics is not only determined by physical constraints induced by other pedestrians, but also significantly by social and communicative interactions among individuals. Finally, we relied on concepts from psychology and cognitive science to elaborate a new modeling framework based on heuristics. This work suggests a crucial paradigm shift from physics-inspired models of pedestrian behaviour toward a cognitive-science approach.

One fundamental element highlighted in this work is the distinction between different *natures* of interactions amon pedestrians. Here we specifically focused on avoidance, social and physical interactions. Many other kinds of interaction could influence pedestrian behaviour as well. For example, when pedestrians tend to escape in the same direction as their neighbours during panics, *imitative* interactions take place (Helbing *et al.*, 2000). In this case, individuals get new information by observing others, such as people fleeing away indicating the presence of a danger. Another kind of interaction occurs when pedestrians walking in a grassy or snowy area tend to follow the trails left by previous walkers (Helbing, 1997; Goldstone & Roberts, 2006). Here the information exchanged among individuals is temporarily printed in the environment. Previous works have shown such *indirect* interactions can generate optimal trail networks in open natural spaces.

Finally, this work shows the importance of interdisciplinary research for understanding crowd dynamics. First, the crowd is a self-organized system as well as animal swarms. Therefore, various theoretical and methodological principles existing in the study of animal behaviour can be transfered to pedestrians, as demonstrated by our experimental approach. At the same time, the crowd is a social system, as shown by the existence of social conventions, and by the communicative features of social interactions among group members. Moreover, we have shown that the crowd is also a physical system as demonstrated by the crucial role of physical interactions in the emergence of crowd turbulence. The physics of crowd motion is particularly relevant at high densities, where individual decisions are increasingly replaced by mass motion, providing good analogies with fluid dynamics. Finally, crowd dynamics has a lot to do with cognitive science, not only regarding the cognitive heuristics used by pedestrians, but also at the collective level when exploring the mecanisms underlying the wisdom of crowds.

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